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On the stability of incompressible viscous fluid motions past objects

by Kyûya MASUDA (Dept. of Math.,Univ. of Tokyo, Komaba)

Let $\mathcal{E}$ be the exterior domain in 3-space. Let us consider the steady flow in $\mathcal{E}$ governed by

$$(1) \qquad \begin{cases} -\nu \Delta w + (w\cdot\nabla)w + \nabla p_1 = 0, \\ \operatorname{div} w = 0 \end{cases}$$

$$(2) \qquad w(x) \longrightarrow w^{\infty} \qquad (|x| \longrightarrow \infty)$$

$$(3) \qquad w(x) = b(x) \qquad (x \in \partial\mathcal{E})$$

where the viscousity coefficient ν is a positive constant, w^{∞} is some fixed constant vector, b is some prescribed function on $\mathcal{E}$.

R. Finn showed that if $w^{\infty} - b$ is"small" enough, then there exists a smooth solution w with

$$\sup_{x\in\mathcal{E}} |x|\,|w(x) - w^{\infty}| < \infty$$

$$\nabla w \in L^3(\mathcal{E})$$

Given the disturbance u_0 ($\in L^2(\mathcal{E})$) to w. Then the perturbed flow v is gorvened by

$$
(4) \quad \begin{cases} \dfrac{\partial v}{\partial t} - \nu \Delta v + (v \cdot \nabla) v - \nabla p_2 = 0 \\ \operatorname{div} v = 0 \end{cases}
$$

$$
(5) \quad \begin{cases} \lim\limits_{|x| \to \infty} v(x,t) = w^\infty, \qquad v(x,t) = b(x) \quad (x \in \partial\mathcal{E}, t > 0) \\ \lim\limits_{t \downarrow 0} v(x,t) = w(x) + a_0(x) \end{cases}
$$

Now our result is:

Assume that

(i) $\quad \sup\limits_{x \in \mathcal{E}} |x| \, |w(x) - w^\infty| < \frac{1}{2}$

(ii) $\quad \nabla w \in L^3(\mathcal{E})$

(iii) $\quad \operatorname{div} w = 0$.

Then evry weak solution v of (4), (5) becomes analytic (in t and x)after some definite time T_0 , and then converges to steady flow w uniformly in x on $\mathcal{E}$ like

$$
|v(x,t) - w(x)| \leq M t^{-1/8}, \quad (t \to \infty)
$$

(M ; constant)